Philosophical methodology and deontic logic

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Non-optional $\varphi$

Obligatory $\varphi$  Forbidden $\varphi$

Permitted $\varphi$  Gratuitous $\varphi$

Optional $\varphi$
I am inclined to doubt whether any special “logic of empirical sciences”, as opposed to logic in general, or, to the “logic of deductive sciences”, exists at all (at least so long as the word “logic” is used as in the present book—that is to say, as the name of a discipline which analyzes the meaning of the concepts common to all sciences, and establishes general laws governing these concepts).

Alfred Tarski. 
*Introduction to Logic and Methodology of Deductive Sciences*, 1. ed. in 1941.

**Vocabulary of philosophy and of sciences of man**

Verbs of belief, desire, intention, action, ability and duty, temporal quantifiers, verb tenses, modal adverbs, non-indicative sentence moods do not appear in the language of any science. The development of philosophical logic in the second half of 20th forces us to widen Tarski’s notion of logic. The rich variety of logical theories (e.g. doxastic logic, bouletic logic, BDI logic, action logic, logic of ability, temporal logic, tense logic, imperative logic, interrogative logic) shows that logic deals not only with “concepts common to all sciences” but also with concepts not common to them all.
Language of philosophy

- The concepts of intentionality (from belief to action) and normativity are essential part of the language of philosophy.
- In some historically influential cases imperatives summarize philosophical theories and world-views.
- The logic of philosophical language cannot be revealed in first-order logic since first-order logic is the theory on the vocabulary common to all sciences: truth-functional connectives (e.g. $\neg$, $\land$, $\lor$, $\rightarrow$, ...), two quantifiers ($\forall$, $\exists$), and identity predicate ($\equiv$).

Some famous imperatives of philosophy

Know yourself! —(Delphic inscription)

Act as if the maxim of your action were to become through your will a universal law of nature. —Immanuel Kant

The philosophers have only interpreted the world, in various ways; the point is to change it. —Karl Marx

You should become who you are. —Friedrich Nietzsche

What we cannot speak about we must pass over in silence. —Ludwig Wittgenstein
One world and one relation between world and language

The philosophy of first-order logic has been exposed in Wittgenstein’s *Tractatus logico-philosophicus*.

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*Tractarian theory*

<table>
<thead>
<tr>
<th>The world</th>
<th>Natural science</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓ Fact</td>
<td>↑ True proposition</td>
</tr>
<tr>
<td>↓ State of affairs</td>
<td>↑ Proposition</td>
</tr>
<tr>
<td>↓ Relation of objects</td>
<td>↑ Predicate and names</td>
</tr>
</tbody>
</table>

▲ shows deconstruction path.
↑ shows construction path.
Is Tractarian approach adequate?

- The understanding of the logic of the language of philosophy and of science of man asks for a different approach: it requires non-Tractarian notions of language and its relation to reality, and it requires a non-Tarskian notion of logic.

- The basic phenomenon in the ontology of social reality is not given by a collection of objects standing in a relation. Rather, the basic phenomena of social reality are constituted by intentionality, individual and collective, and normativity. Social facts and physical facts are different in category, and it is not surprising that the logics of their respective languages are not the same.

- Next we turn to philosophy of science of man in order to determine its specific logical terminology and characteristic theoretical constructions.
Two kinds of sciences

- Wilhelm Dilthey (1833–1911) gave epistemological explication of the difference between two kinds of science:
  - humanities and social sciences, and
  - natural sciences.

The first ones aim to understanding meaning of individual acts, the second ones seek general laws covering natural events.

- Wilhelm Windelband (1848–1915) coined the adequate names
  - idiographic sciences, and
  - nomothetic sciences.

- Donald Davidson (1917–2003) pointed out differences of the languages they employ both in terms of their vocabulary and logic: the language of former creates “intensional contexts” which have no place in the language of the latter.

The vocabularies together with their transformational syntax are termed ‘mental’ and ‘physical.’

- actions, reasons, persons, lived experiences, . . .
- events, causes, things, states of affairs, . . .
Von Wright’s methodological thesis

Practical syllogism grounds methodological autonomy of sciences of man

Practical reasoning is of great importance to the explanation and understanding of action. It is a tenet of the present work that the practical syllogism provides the sciences of man with something long missing from their methodology: an explanation model in its own right which is a definite alternative to the subsumption-theoretic covering law model. Broadly speaking, what the subsumption-theoretic model is to causal explanation and explanation in the natural sciences, the practical syllogism is to teleological explanation and explanation in history and the social sciences.

What is practical syllogism?

- Aristotle discovered practical inference as different in kind from the theoretical (cf. e.g. *Nicomachean ethics* 1112b, 1147b; *Metaphysics* 1032b, *De Motu Animalium* 701a). Their conclusions answer to different questions
  - practical what to do?
  - theoretical what is the case?

**Important but neglected**

‘Practical reasoning,’ or ‘practical syllogism,’ which means the same thing, is one of Aristotle’s best discoveries. But its true character has been obscured.

An exemplar of practical inference

Practical inference

\[ \begin{align*} A \text{ intends to bring about } p. \\
A \text{ considers that he cannot bring about } p \text{ unless he does } a. \\
\text{Therefore } A \text{ sets himself to do } a. \end{align*} \]

A schema of this kind is sometimes called a practical inference (or syllogism). I shall use this name for it here, without pretending that it is historically adequate, and consciously ignoring the fact that there are many different schemas which may be grouped under the same heading.

A rough analysis

We find at least four expressions that invoke modal logic treatment:

“intentionality modalities” :

praxeologic modality  [A brings it about that], [A does so that],
[A sets himself to do]

bouletic modality  [A intends to]

doxastic modality  [A considers that]

alethic modality  ⟨it is possible that⟩ for ‘can’
Practical inference in a simplified form

Practical inference is usually understood as exemplar form of teleological explanation: agent A’s action \( a \) is teleologically explained in terms of agent’s intention (\([I_A]\)), whose content is the goal \( p \), and agent’s belief (\([B_A]\)) that agent’s doing \( a \) is necessary for the realization of intended goal \( p \).

\[
\begin{array}{c}
[I_A] & p \\
[B_A] & (\Diamond p \rightarrow [Do_A] a) \\
[Do_A] & a
\end{array}
\]

Notice that if all four modal operators are erased, then we get *modus ponendo ponens*.

Practical inference belongs to the realm of intentionality. But the logic of intentional states is not clear even for single modalities, let alone their combinations. In that respect, one can repeat Anscombe’s words: the true character of the logic of intentionality is still obscure.

\[^1p \text{ is possible (\Diamond) only if } [Do_A] a).\]
One more example

Example (An exemplar philosophical sentence)

When someone believes she ought to do something, often her belief causes her to intend to do it.

- Due to the fact that they are expressed in natural language, philosophical sentences sound familiar and easy to understand, but the impression is deceptive.

- Let us extract the logical elements form the exemplar sentences! There are (with contextual disambiguation written within parentheses): (i) temporal quantifiers: often (the number of occurrences of a phenomenon is at least as great as the number of its non-occurrences), (ii) persons quantifier: someone (anybody), (iii) doxastic modality: belief, (iv) normative modality: ought, (v) praxeological modality: action, (vi) states of affairs quantifier (something), (vii) causality relation, (viii) bouletic modality: intention.

- At present no logical system is capable of accommodating all of these elements.
The modal approach to logical syntax

Example

1. Actor i believes that there are abstract objects.
   Let \( B_i \) stand for ‘Actor i believes that’ and \( p \) for ‘there are abstract objects’.
   1. translates to: \( B_i p \)
      No matter whether \( p \) is true or not, \( B_i p \) can be true since it is an assertion about i’s belief and not about abstract objects.

2. Actor i ought to believe that there are abstract objects.
   Let \( O_i \) stand for ‘It is obligatory for i that’.
   2. translates to: \( O_i B_i p \)
      No matter whether \( B_i p \) is true or not, \( O_i B_i p \) can be true since it is an assertion about i’s obligation and not about i’s belief.

3. Actor i believes that she ought to believe that there are abstract objects.
   3. translates to: \( B_i O_i B_i p \)
      No matter whether \( O_i B_i p \) is true or not, \( B_i O_i B_i p \) can be true since it is an assertion about i’s belief (about her obligation to believe \( p \)) and not about i’s obligation (to believe that \( p \)).

\(^a\)E.g. i wants to make friends with j who is a determined Platonist.
\(^b\)E.g. j gladly accepts non-Platonists among her friends.
Modal operators: syntax

- From the syntactical point of view modal operators are similar to connectives: they take apply to sentences and deliver new sentences.
- In particular, modal operators are similar to unary (one-place) sentential operators.

Example

Negation (‘it is not the case that’ or \( \neg \)) is a one-place operator and when applied to a sentence (‘there are abstract objects’ or \( p \)) it yields a new sentence (‘it is not the case that there are abstract objects’ or \( \neg p \)).

Doxastic operator (‘actor i believes that’ or \( B_i \)) is an one-place operator and when applied to a sentence (‘there are abstract objects’ or \( p \)) it yields a new sentence (‘actor i believes that there are abstract objects’ or \( B_ip \)).

- The difference between connectives and modal operators lies in its number: —there is small number of connectives (moreover their number can be reduced to one without loss of expressive power), —there is irreducible abundance of modal operators.
Modal operators: semantics

- From the semantic point of view modal operators are not at all similar to connectives:
  - Connectives are truth-functional: an application of a connective always yields a new sentence whose truth-value is determined by the truth-value of its constituent sentential parts.
  - Modal operators are not truth-functional: an application of a modal operator can yield a new sentence whose truth-value is not determined by the truth-value of its constituent sentential parts.

- Because of non-truth-functional character of modal operators modal logic is also called ‘intensional logic’.

Example (Intensionality of “mental vocabulary”)

Let modal operator $D_{lois}$ stand for ‘Lois desires that’.

1. $D_{lois} \text{Spouse}(lois,\,\text{superman})$
2. $\text{Spouse}(lois,\,\text{superman}) \leftrightarrow \text{Spouse}(lois,\,\text{clark})$
3. $D_{lois} \text{Spouse}(lois,\,\text{clark})$

In the famous comic book by Joseph Shuster (1914–1992) premises are true while conclusion is not.
Mental processes

THREE LOGICIANS WALK INTO A BAR...

DOES EVERYONE WANT BEER?

I DON'T KNOW.

I DON'T KNOW.

YES!

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Mental states

Exercise

Let us analyse the mental process described in the cartoon above! We will identify the belief state of each actor with the set of situation that she considers possible. Let \( p(q, r) \) stand for ‘the first actor (the second actor, the third actor) wants beer’. A possible situation can identified with a valuation. In the beginning every actor knows only her own desires and is ignorant of desires of others, and therefore there are exactly four valuations that each of them considers possible.

An actor’s ignorant answer to the waiter’s question shows that she wants beer (for if she did not want beer her answer to the question ‘Does everybody want beer’ would be ‘No’) and that answer gives information to the other actors who update their belief state accordingly. Let us use calculator at http://www.ffst.hr/~logika/implog/calculators/update/update.html and reconstruct the dynamics of the belief change of the last actor!
One world is not enough

- According to the *Tractarian* criterion, modal propositions are not propositions at all since they are not truth-functions.
- The “one world” semantic theory cannot accommodate modal propositions since the truth value of their “elementary propositions” in *The World* does not determine the truth-value of the modal compound.

**Example**

The truth of what is obligatory to be (ought to be) the case is logically independent of that what is the case. Let $O$ stand for ‘It is obligatory that’. Both $p \land Op$ and $\neg p \land Op$ are satisfiable.

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**Tractatus logico-philosophicus**

5 Propositions are truth-functions of elementary propositions.

... 

6.42 Hence also there can be no ethical propositions.

... 

7 Whereof one cannot speak, thereof one must be silent.
What to do?

The consequence of the “one world” semantics is not just being silent about certain topics, but rather abandonment of the huge part of language.
Leibniz and modal analysis of normative concepts

Modal approach to normativity

Licitum enim est, quod viro bono possibile est.
Debitum sit, quod viro bono necessarium est.\(^a\)

Gottfried Wilhelm Leibniz.
Letter to Antoineu Arnauldu, November 1671.
Berlin: Akademie Verlag.

\(^a\)That is permitted what a good man possibly is.
That is obligatory what a good man necessary is.

In Leibniz’s definition normative concepts (permission \(P\), obligation \(O\)) are defined in terms of (i) alethic modalities (possibility \(\Diamond\), neccessity \(\Box\)) (ii) normative properties (being a good man \(G_i\)).

\[
\begin{align*}
O\varphi & \leftrightarrow \Box(G_i \rightarrow \varphi) \\
P\varphi & \leftrightarrow \Diamond(G_i \land \varphi)
\end{align*}
\]
Deontic logic as modal logic

Philosopher’s recollection

One day when I was walking along the banks of the River Cam—I was at that time living in Cambridge (England)—I was struck by the thought that the modal attributes “possible,” “impossible” and “necessary” are mutually related to one another in the same way as the quantifiers “some,” “no” and “all.” I soon found that the formal analogy between quantifiers and modal concepts extended beyond the patterns of interdefinability... I had made another accidental observation—this time in the course of a discussion with friends—namely that the normative notions of permission, prohibition, and obligation seemed to conform to the same pattern of mutual relatedness as quantifiers and basic modalities.

Georg Henrik von Wright.
Deontic logic: a personal view.
**Analogy of quantification and modality**

**Duality; square of oppositions**

<table>
<thead>
<tr>
<th>Quantifiers</th>
<th>Alethic modalities</th>
<th>Deontic modalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\forall x \varphi \quad (\neg \exists x \neg \varphi)$</td>
<td>$\Box \varphi \quad (\neg \Diamond \neg \varphi)$</td>
<td>$O \varphi \quad (\neg P \neg \varphi)$</td>
</tr>
<tr>
<td>All . . .</td>
<td>NECESSARY . . .</td>
<td>OBLIGATORY . . .</td>
</tr>
<tr>
<td>$\exists x \varphi \quad (\neg \forall x \neg \varphi)$</td>
<td>$\Diamond \varphi \quad (\neg \Box \neg \varphi)$</td>
<td>$P \varphi \quad (\neg O \neg \varphi)$</td>
</tr>
<tr>
<td>Some . . .</td>
<td>POSSIBLE . . .</td>
<td>PERMITTED . . .</td>
</tr>
<tr>
<td>$\forall x \neg \varphi \quad (\neg \exists x \varphi)$</td>
<td>$\neg \varphi \quad (\neg \Diamond \varphi)$</td>
<td>$F \varphi \quad (O \neg \varphi$, i.e. $\neg P \varphi)$</td>
</tr>
<tr>
<td>No . . .</td>
<td>IMPOSSIBLE . . .</td>
<td>FORBIDDEN . . .</td>
</tr>
</tbody>
</table>

**Modal logic**

- **O** (Obligatory)
- **P** (Permitted)
- **F** (Forbiden)

**First-order logic**

- **∀** (For all)
- **∃** (There exists)
Hexagon of “oppositions”

Four logical relations resulting from mutual definability (duality) of normative notions. The last one is $D$ axiom

<table>
<thead>
<tr>
<th>Name</th>
<th>Property</th>
<th>Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contrariety</td>
<td>Both sentences cannot be true.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Subcontrariety</td>
<td>Both sentences cannot be false.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Contradiction</td>
<td>Both sentences cannot be true, and both sentences cannot be false.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Implication</td>
<td>It cannot be so that the source sentence is true and target sentence is false. No.</td>
<td></td>
</tr>
<tr>
<td>Implication</td>
<td>It cannot be so that the source sentence is true and target sentence is false. No.</td>
<td></td>
</tr>
</tbody>
</table>

$^2$Some synonyms:[Permitted; Allowed][Optional; Allowed and non-obligatory][Gratuitous; Non-obligatory; Omissible][Forbidden; Prohibited; Impermissible][Non-optional; Obligatory or forbidden][Obligatory]
D axiom

Obligatory $O\varphi \leftrightarrow \neg P\neg \varphi \leftrightarrow F\neg \varphi$

Forbidden $F\varphi \leftrightarrow \neg P\varphi \leftrightarrow O\neg \varphi$

The “black arrow” implications $O\varphi \rightarrow P\varphi$ and $F\varphi \rightarrow P\neg \varphi$ are equivalent. $^a$ This implication is called D axiom. It can also be read as $\neg(O\varphi \land F\varphi)$, i.e., as a claim on contrariety of $O\varphi$ and $F\varphi$.

The analogy with alethic modalities holds since $\Box \varphi \rightarrow \Diamond \varphi$.

$^a$Assuming modal congruence.
Axioms and rules of standard deontic logic

Standard deontic logic $\text{KD}$ is a normal logic, which means that it provides:

- **K axiom (schema):**
  \[ O(\varphi \rightarrow \psi) \rightarrow (O\varphi \rightarrow O\psi) \]

- **RN necessitation rule:**
  
  If $\vdash \varphi$, then $\vdash O\varphi$.

Rule RN and axiom K define the character of modal possibilities: they obey the rules of logic (and therefore are ‘normal’). By RN, logical truths hold in any deontic possibility. By K, the consequences of truths of a deontic possibility are the truths of it.

The only additional axiom of deontic logic is:

- **D axiom (schema):**
  \[ O\varphi \rightarrow P\varphi \]

Rule RN can be deontically interpreted as “permission implies logical possibility”.³

Deontic interpretation of K is “logical consequences of obligations are obligations themselves”.

Axiom D roughly translates to “it is permitted to fulfil an obligation”.

³Equating provability without premises with logical necessity RN becomes $\Box \varphi \rightarrow O\varphi$, and conversion gives suggested reading.
Example

The Roman Law principle *ultra posse nemo obligatur* is also known as *ought implies can* principle, and usually mistakenly attributed to Kant. The principle is translated here in its simplified form (Proposition below): (i) standard deontic logic deals with that which *ought to be* and not, as a full-blown deontic logic should, with that which *ought to be done*, (ii) the alethic modality of logical possibility will be used instead of ability modality.

Lemma

\[ \vdash P\varphi \rightarrow \diamond \varphi \]

Proposition

\[ \vdash O\varphi \rightarrow \diamond \varphi \text{ (i.e., } \vdash \neg \diamond \varphi \rightarrow \neg O\varphi \text{).} \]

Proof.

1. Assume \( O\varphi \).
2. \( P\varphi \), from (1) by D.
3. \( \diamond \varphi \), from (2) by lemma.
4. Therefore, \( \vdash O\varphi \rightarrow \diamond \varphi \).
Unexpected results

- The introduction of relational semantics (“possible world semantics”, simultaneously and independently discovered in late 1950s by Stig Kanger and Saul Kripke) has brought some amazing insights in philosophy.

- The analogy between quantification, on the one side, and alethic and deontic modality, on the other side, has received its formal semantic explanation.
Many worlds and their relations

- **Many worlds.** Modal expressions involve hidden quantification:
  1. Some modalities are universal, like □ or O, and they talk about *all* possibilities (valuations, states, possible worlds) within the appropriate category (logical, deontic,...),
  2. Some modalities are existential, like ♦ or P, and they in the similar manner talk about *some* possibilities.

- **Structure.** The plurality of valuations is not sufficient. The distinction between modalities having the same quantificational character but validating different principles has been found in the way the possibilities are connected. The possibilities to be taken into account at the point of evaluation. Quantifiers ∀ and ∃ offer “bird’s eye view”: their perspective is global and covers all objects. Modalities give a local picture, a “frog’s eye view”: their perspective is located at a particular evaluation point (“the point where we stand”) and therefore covers all possibilities accessible (“visible”) from that point.

**Example**

Alethic logic readily accepts the principle of existential modal generalization: ‘if something is the case, that it is possible’ or \( \varphi \rightarrow \Diamond \varphi \). Deontic logic readily rejects that principle: it is not valid to claim that ‘if something is the case, that it is permitted’ or \( \varphi \rightarrow P \varphi \).
Modal calculator:
http://www.ffst.hr/~logika/implog/calculators/modal/modal.html
Instructions:
http://www.ffst.hr/~logika/implog/doku.php?id=program:possible_worlds
A new language

The researchers in philosophical logic came upon an amazing insight: for modal axioms there are corresponding properties of accessibility relation. (Axiom K and rule RN are different in category: they define the character of the worlds and say nothing about their connections.) Let’s try to introduce this insight by way of a metaphor!

A metaphor

Imagine yourself being repeatedly placed in one world after another within a network of worlds. You have an axiom map: a sentential form that must come out true no matter which sentences you put into it. Your “positive task” is to test the accuracy of the map in the modal way: by looking at accessible worlds, possibly moving there and looking at accessible worlds from there, and possibly repeating the action again but in finitely number of times. It turns out that you come up with positive test results for each of successive placements. After that, you have an additional, more complicated test called “negative task”: after being placed in a world you have to investigate whether it is possible to modify the world you are at and the worlds accessible from it so to make the axiom map false. If the positive task always results in affirmative (map is true) and the negative task always gives the negative answer (it is not possible to modify worlds so to falsify the map), then your axiom map is accurate and it describes some property of the paths connecting the worlds. E.g. if the map $\Box \varphi \rightarrow \Box \Box \varphi$ passes both tests in a network of worlds, i.e. if is accurate, then the following fact on the property of paths holds: if you can get from the source world to the target world via an intermediate one, then you can also get directly from the source to the target.
The correspondence between axioms and properties of accessibility relations has revealed an important characteristics of the logic of the language of philosophy and science of man.

Modal logic is not just another way to define implicitly modal terms by fixing their meaning in axioms. Rather, it is a discovery of a language. The language of propositional modal logic turns out to have high expressive power, different in kind from that of the language of propositional logic but lower in discriminatory power from the first-order language.

The “geometry of meaning” extends far beyond the square or hexagon of oppositions: the logical “space” of modal operators is structured so that different structural types correspond to different modality types.

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4. While the language of propositional logic has no discriminatory power, the language of propositional modal logic can discriminate between finite structures up to bisimilarity. The language of first-order logic can discriminate between finite structures up to isomorphism (a type of “picture relation” stronger than bisimilarity).
Extensional vs. intensional semantics

- In extensional semantics the truth-value of a compound sentence depends on the truth values of its constituent sentences and the rule of valuation associated with the connective. E.g. the rule associated with $\wedge$ determines the truth-value of $\varphi \wedge \psi$ on the basis of truth-values of $\varphi$ and $\psi$.

- Although there is a rule associated to modal operators, the truth-value of the compound sentence whose main operator is modal operator cannot be determined on the basis of the truth values of its constituent sentences. The reason for that is “hidden quantification”: rules associated with modal operators determine the truth-value of the compound by taking into account truth-value of its constituent at all (accessible) points of valuation. E.g. the rule associated with $\Box$ determines the truth-value of $\Box\varphi$ on the basis of truth-value of $\varphi$ at each accessible point: $\Box\varphi$ is true in a world $w$ if $\varphi$ is true at each point (each deontic possibility, each world where that what ought to be is the fact of that world) accessible from $w$.

- The semantics that takes into account multiple valuations of the same syntactic object is called intensional semantics.
Geometry of meaning of modal operators

- Intensional semantics takes into account multiple valuations but the meaning of modal operators is not reducible to them. If it were, we could not distinguish the types of operators since the same rule would be associated with deontic $\Box$, alethic $\square$, epistemic $K$, as well as any other universal modal operator.

- The geometry of meaning of modal operators is given by the properties of accessibility relation. Different modalities have different types of accessibility. The type of accessibility is determined by modal axioms. E.g. what property must the relation of deontic accessibility have? The one that corresponds to the meaning of deontic operators, and that meaning is fixed by axioms.

- Thus, the meaning of modal words has two parts:
  1. **Rule.** Quantification part. Common to all modality types.
  2. **Structure.** Space of quantification. Specific for given modality type. Corresponds to axioms of some regional modal logic, and, therefore, exhibits the “geometry of meaning” of the particular modal word.